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THE CONTACT PROBLEM OF THE WEAR CAUSED BY MELTING OF THE BUSHING OF A SLIDING BEARING†

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The contact problem of the wear caused by the melting of a thin polymer bushing of a sliding bearing is considered on the assumption that the coefficient of friction is a function of temperature. The relation for the critical speed of rotation of the shaft at which melting begins is obtained. Formulae for the basic parameters of the contact interaction such as the residue of the bushing, the angle of contact and the contact pressure are also derived.

THE PLANE and axisymmetric contact problems of the wear caused by the melting of an elastic layer and a half-space was first stated and solved in [1, 2] on the assumption that the contact area does not vary with time and that, because of friction forces, the rise in temperature under a punch is sufficient to melt the elastic base. In this case the melted material was pressed out from the region under the punch, and caused it to be deposited. In what follows, a method of solving the contact problem of the wear caused by melting is presented for the case of a variable contact area.

1. Consider a thin polymer ring (that is the bushing of a sliding bearing) of inner radius R and of outer radius R. The ring is connected to a non-deformable race along the outer contour, and a part of its inner surface is in a contact with a rigid shaft of radius $R_1 = R - \Delta$ $(\Delta R^{-1} \ll 1,$ $hR^{-1} \ll 1,$ $h = R_2 - R)$ which rotates about its axis with a constant angular velocity ω and produces a force $P = P(t)$ on the bushing (Fig. 1). Wear of the surface of the bushing occurs during the process, and is accompanied by the generation of heat in the contact area. Moreover, if $\omega < \omega_*$, the wear is caused by fretting [3] but when $\omega > \omega_*$ the wear starts as a result of melting (ω_* is the critical value of the rotation speed of the shaft, and will be specified below). We will restrict ourselves here by examining the second situation when $\omega > \omega_*$.

Assume that (1) the shaft and the race have no wear, i.e. their melting points are far higher than that of the ring, (2) the viscosity of the bushing can be neglected, (3) inertial effects of the ring can be negligibly small, and (4) the friction force is related to the contact pressure $q(\varphi, t)$ by the Coulomb law with the coefficient depending on the temperature $T = T(\varphi, t)$ in the contact area $|\varphi| \leqslant \alpha(t)$

$$\tau_{r\varphi} = \tau(\varphi, t) = f(T)\, q(\varphi, t) \tag{1.1}$$

From now on, the temperature of the surrounding medium is taken as the origin of the temperature scale, and the subscripts 1, 2 and 3 are ascribed to quantities relating to the shaft, ring and race, respectively.

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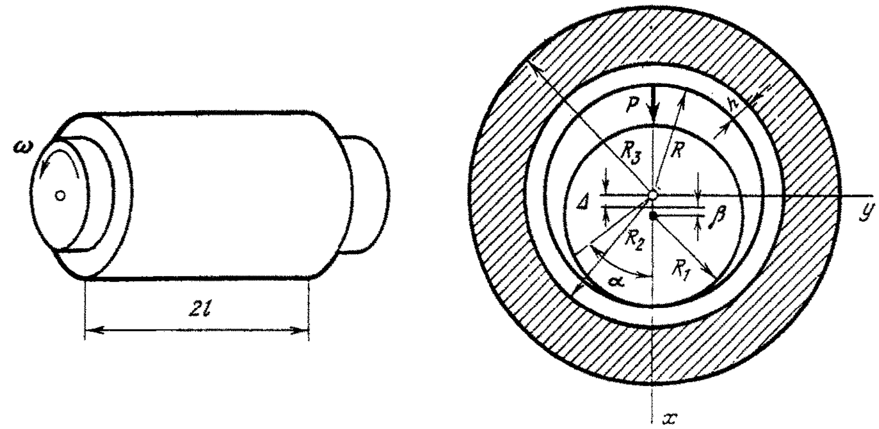

FIG. 1.

We will assume that the force $P(t)$ varies in time in such a manner that the function $\alpha = \alpha(t)$ increases monotonically with time. In this case, the inverse function $t = \eta(\alpha)$ exists, and since it is single-valued, and, it is possible to use α as a new time parameter [4, 5]. Therefore let us assume that the dependence of the corresponding quantities on time is complex, for instance, $q(\varphi,\ t) = q(\varphi,\ \alpha(t))$, $P(t) = P(\alpha(t))$, etc.

We will write the equation of the energy balance [2] when $\omega > \omega_*$

$$Q(\varphi, t) = Q_1(t) + Q_2(t) + Q_0(\varphi, t) \quad (|\varphi| \leqslant \alpha) \tag{1.2}$$

In (1.2) $Q(\varphi, t)$ is the total amount of heat generated per unit of time, it is proportional to the power of friction forces

$$Q(\varphi, t) = \omega R_1 \tau(\varphi, t) \tag{1.3}$$

$Q_2(t)$ is the value computed by averaging the heat flux into the bushing over the contact area (the heat flux maintains the temperature in this area near the melting point T_* of the material of the bushing), $Q_1(t)$ is the similar value of the heat flux into the shaft, and $Q_0(\varphi,\ t)$ is the amount of heat required to melt the material of the ring when $|\varphi| \leqslant \alpha(t)$.

We will assume that the condition of the contact of the shaft with the bushing has the form

$$u(\varphi, t) + v(\varphi, t) = [\beta(t) + \Delta] \cos\varphi - \Delta \quad (|\varphi| \leqslant \alpha) \tag{1.4}$$

where $u(\varphi, t)$ are the thermoelastic radial displacements of the points of the ring, $v(\varphi, t)$ are the displacements caused by its wear, and $\beta(t)$ is a translational displacement of the tenon under the action of the force P.

In order to evaluate the function $u(\varphi, t)$ we will change the dimensionless variable

$$r = Re^{\epsilon\rho}, \quad \epsilon = \ln(1 + h/R) \tag{1.5}$$

in Lamé's equations with temperature terms and in the formulae of Hooke's law.

We then have

$$\left(\frac{\partial^2}{\partial\rho^2} - \epsilon^2 + \kappa\epsilon^2 \frac{\partial^2}{\partial\varphi^2}\right) u_r + \frac{\partial}{\partial\varphi}\left[(1-\kappa)\epsilon \frac{\partial}{\partial\rho} + (1+\kappa)\epsilon^2\right] u_\varphi - BR\epsilon e^{\epsilon\rho} \frac{\partial T_2}{\partial\rho} = 0 \tag{1.6}$$

$$\left(\frac{\partial^2}{\partial\rho^2} - \epsilon^2 + \frac{\epsilon^2}{\kappa} \frac{\partial^2}{\partial\varphi^2}\right) u_\varphi + \frac{1}{\kappa} \frac{\partial}{\partial\varphi}\left[(1-\kappa)\epsilon \frac{\partial}{\partial\rho} + (1+\kappa)\epsilon^2\right] u_r -$$

$$-\frac{B}{\kappa}R\epsilon^2 e^{\epsilon\rho}\frac{\partial T_2}{\partial\varphi}=0\quad(\kappa=\frac{1-2\nu}{2(1-\nu)},\ B=\frac{\alpha_T(1+\nu)}{1-\nu})$$

$$\tau_{r\varphi}=\frac{G}{R}e^{-\epsilon\rho}(\frac{\partial u_r}{\partial\varphi}-u_\varphi+\frac{1}{\epsilon}\frac{\partial u_\varphi}{\partial\rho})$$

$$\sigma_r=\frac{G}{\kappa R}e^{-\epsilon\rho}[\frac{1}{\epsilon}\frac{\partial u_r}{\partial\rho}+(1-2\kappa)(u_r+\frac{\partial u_\varphi}{\partial\varphi})]-\frac{BG}{\kappa}T_2 \tag{1.7}$$

Here u_r and u_φ are the radial and tangential components of the vector of displacements of the points of the ring, σ_r and $\tau_{r\varphi}$ are the corresponding components of the stress tensor, G and ν are the elastic parameters of the bushing, T_2 is its temperature, and α_T is the coefficient of linear expansion.

We will construct a solution of system (1.6) which is asymptotic to within $O(\epsilon^2)$ and satisfies the equalities

$$\rho=1:u_r=u_\varphi=0 \tag{1.8}$$

$$\rho=0:\tau_{r\varphi}=\tau(\varphi,t),\ \sigma_r=-q(\varphi,t)$$

For that purpose we express the functions u_r and u_φ in the form

$$u_r=\Phi_0(\varphi,\rho,t)+\epsilon\Phi_1(\varphi,\rho,t)+O(\epsilon^2) \tag{1.9}$$

$$u_\varphi=\Psi_0(\varphi,\rho,t)+O(\epsilon)$$

If we substitute formulae (1.9) into relations (1.6)–(1.8) and simplify the result making use of the asymptotic forms, we obtain the following recurrent system

$$\frac{\partial^2\Phi_0}{\partial\rho^2}=0,\ \frac{\partial^2\Phi_1}{\partial\rho^2}=0,\ \frac{\partial^2\Phi_1}{\partial\rho^2}+(1-\kappa)\frac{\partial^2\Psi_0}{\partial\rho\,\partial\varphi}=BR\frac{\partial T_2}{\partial\rho} \tag{1.10}$$

with boundary conditions

$$\rho=1:\Phi_0=\Psi_0=\Phi_1=0$$

$$\rho=0:\frac{\partial\Phi_0}{\partial\rho}=-\epsilon\frac{R\kappa}{G}q,\ \frac{\partial\Psi_0}{\partial\rho}=\epsilon\frac{R}{G}\tau \tag{1.11}$$

$$\frac{\partial\Phi_1}{\partial\rho}+(1-2\kappa)(\Phi_0+\frac{\partial\Psi_0}{\partial\varphi})=BRT_2$$

The solutions of Eqs (1.10) and (1.11) have the form

$$\Phi_0=-\epsilon\frac{R\kappa}{G}(\rho-1)q,\ \Psi_0=\epsilon\frac{R}{G}(\rho-1)\tau \tag{1.12}$$

$$\Phi_1=BR(\rho-1)T_2-\epsilon\frac{(1-\kappa)R}{2G}\rho^2\frac{\partial\tau}{\partial\varphi}+\epsilon\frac{(1-2\kappa)R}{G}[\rho\frac{\partial\tau}{\partial\varphi}-$$

$$- \kappa (\rho - 1) q] - \epsilon \frac{(1 - 3\kappa) R}{2G} \frac{\partial \tau}{\partial \varphi}$$

If we substitute (1.12) into the first formula of (1.9) and set $\rho = 0$ in the relation obtained, we obtain

$$u_r(0, \varphi, t) = u(\varphi, t) = \epsilon(\kappa R G^{-1} q - BRT) +$$
$$+ \epsilon^2 \frac{R}{G} [\kappa(1 - 2\kappa) q - \frac{1}{2} (1 - 3\kappa) \frac{\partial \tau}{\partial \varphi}] \quad (|\varphi| \leqslant \alpha) \tag{1.13}$$

Note that expression (1.13) was obtained in [6] in a more complicated manner. Henceforth, while using formula (1.13), we shall neglect the second term in it since it is much smaller than the first one.

2. In order to find the displacement $v(\varphi, t)$ let us evaluate the critical speed ω_* by solving first the steady-state problem of heat conduction for the ring

$$\nabla^2 T_2 = 0 \quad (\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2}) \tag{2.1}$$

$$r = R : T_2 = T_* \quad (|\varphi| \leqslant \alpha) \tag{2.2}$$
$$- \lambda_2 \frac{\partial T_2}{\partial r} + \kappa_2 (T_2 - T_a) = 0 \quad (|\varphi| > \alpha)$$
$$r = R_2 : T_2 = \Phi(\varphi) \quad (|\varphi| \leqslant \pi)$$

Here λ_2 and k_2 are, respectively, the coefficients of thermal conductivity and heat exchange coefficient for the material of the bushing, and T_a is the temperature of the air in the bearing.

In accordance with the physical meaning of the original problem we have the condition $\lambda_2 \lambda_3^{-1} \ll 1$ (λ_3 is the coefficient of thermal conduction of the race) in consequence of which the projection of the temperature gradient onto the direction from the race to the bushing is small. For this reason we assume that the temperature distribution on the outer surface of the bushing is specified.

In order to evaluate the function $\Phi(\varphi)$ we should study the problem on heat conduction for the race. From a consideration of the data on the extrema of the temperature distribution on its inner surface we can conclude that this distribution is governed by the relation [7]

$$T_2(R_2, \varphi) = T_3(R_2, \varphi) = \Phi(\varphi) = D_0 + D_1 \cos \varphi \tag{2.3}$$

We find the constants D_0 and D_1 by making use of the measurements data of the temperature at two characteristic points of the contact area of the race with the bushing for each instant of functioning of the bearing

$$D_0 = \frac{T_{2\,\max} + T_{2\,\min}}{2}, \quad D_1 = \frac{T_{2\,\max} - T_{2\,\min}}{2}$$

$$T_{2\,\max} = T_2(R_2, 0), \quad T_{2\,\min} = T_2(R_2, \pi)$$

The solution of the boundary-value problem (2.1), (2.2) may be constructed by the asymptotic method given in Sec. 1, where the problem of thermoelasticity for a thin ring was examined, if we introduce the dimensionless variable ρ of form (1.5) into the original expressions. Omitting details we write

$$T_2 = T_* - (T_* - \Phi)\rho \quad (|\varphi| \leqslant \alpha)$$

$$T_2 = \frac{\epsilon\kappa_2 R(\Phi - T_a)}{\lambda_2 + \epsilon\kappa_2 R}\rho + \frac{\lambda_2\Phi + \epsilon\kappa_2 R T_a}{\lambda_2 + \epsilon\kappa_2 R} \quad (|\varphi| > \alpha) \tag{2.4}$$

as a first approximation.

Using of the first formula of (2.4) we evaluate the heat flux $-\lambda_2 \partial T_2/\partial r$ when $r = R$, by averaging this function over the contact area, and taking account of relation (2.3), we obtain

$$Q_2(t) = \lambda_2(\epsilon R)^{-1}(T_* - D_0 - D_1\alpha^{-1}\sin\alpha) \tag{2.5}$$

Now let us find the value $Q_1(t)$. For this purpose we solve the problem of heat conduction for an infinite shaft assuming that the bushing is of finite length $2l$. We stress [8] that when the rotation speed is greater than 5 sec^{-1}, the assumption of a uniform distribution of the incoming heat flux along the entire surface of the shaft is valid. By virtue of this, it is possible to study the problem according to the simplified axisymmetric statement using the averaged temperature of the shaft

$$\overline{T}_1(r, z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} T_1(r, \varphi, z)\, d\varphi \tag{2.6}$$

We then have

$$\frac{\partial^2 \overline{T}_1}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}_1}{\partial r} + \frac{\partial^2 \overline{T}_1}{\partial z^2} = 0 \tag{2.7}$$

$$r = R_1 : \overline{T}_1 = T_* \quad (|z| \leqslant l)$$
$$\lambda_1 \partial\overline{T}_1/\partial r + \kappa_1 \overline{T}_1 = 0 \quad (|z| > l), \quad \overline{T}_1 \to 0 \quad (|z| \to \infty) \tag{2.8}$$

To solve problem (2.7), (2.8) let us employ the Fourier transform with respect to the variable $z = xl$. We obtain an integral equation of the first kind

$$\int_{-1}^{1} k\left(\frac{\xi - x}{\mu}\right) p(\xi)\, d\xi = \pi\frac{\lambda_1}{l}T_* \quad (|x| \leqslant 1) \tag{2.9}$$

$$k(y) = \int_0^\infty \frac{I_0(u)\cos uy\, du}{uI_1(u) + \lambda I_0(u)} \quad \left(\lambda = \frac{\kappa_1 R_1}{\lambda_1},\ \mu = \frac{R_1}{l}\right)$$

for the function $p(x)$ defined by the relation

$$\lambda_1\frac{\partial \overline{T}_1}{\partial r} + \kappa_1\overline{T}_1 = \begin{cases} p(x) & (r = R_1,\ |x| \leqslant 1) \\ 0 & (r = R_1,\ |x| > 1) \end{cases} \tag{2.10}$$

In practice, the parameter λ varies over the interval $0 < \lambda \leqslant 0.3$, as a rule. Taking this fact into account we can approximate the symbol of the kernel in the integral equation (2.9) by the expression [9]

$$\frac{I_0(u)}{uI_1(u) + \lambda I_0(u)} \approx \frac{c_2 + c_3u^2}{u^4 + (c_1 + \lambda c_3)u^2 + \lambda c_2} \tag{2.11}$$

$$c_1 = 38.37;\ c_2 = 76.74;\ c_3 = 11.45$$

to within 3%.

Note that Eq. (2.9) with (2.11) is correctly solvable in the space of generalized slowly increasing functions and its solution is represented in the form [10]

$$p(x) = A_1 + A_2 \operatorname{ch}\left(\frac{sx}{\mu}\right) + A_3\left[\delta\left(\frac{x+1}{\mu}\right) + \delta\left(\frac{x-1}{\mu}\right)\right],\quad s = \sqrt{\frac{c_2}{c_3}} \tag{2.12}$$

$$A_1 = \kappa_1 T_*,\ A_2 = -\Lambda^{-1} A_1 (\zeta_1^2 - s^2)(\zeta_2^2 - s^2)$$

$$A_3 = A_1 \{\zeta_1^{-1} - \Lambda^{-1}(\zeta_2^2 - s^2)[\zeta_1 \operatorname{ch}(s/\mu) + s \operatorname{sh}(s/\mu)]\}$$

$$\Lambda = \zeta_1\zeta_2[(s^2 + \zeta_1\zeta_2)\operatorname{ch}(s/\mu) + (\zeta_1 + \zeta_2) s \operatorname{sh}(s/\mu)]$$

$$\zeta_{1,2} = \left\{\frac{c_1 + \lambda c_3}{2} \pm \left[\frac{(c_1 + \lambda c_3)^2}{4} - \lambda c_2\right]^{1/2}\right\}^{1/2}$$

where $\delta(t)$ is the Dirac delta function. When we average relation (2.12) with respect to x and substitute the result into (2.10) we obtain

$$Q_1 = \lambda_1 \partial \bar{T}_1/\partial r = \mu[A_2 s^{-1} \operatorname{sh}(s/\mu) + A_3] \tag{2.13}$$

if the first condition of (2.8) is taken into account.

To determine the critical speed of rotation of the shaft $\omega_c = \omega_c(\alpha)$ at which the melting of the material of the bushing begins, let us approximate the contact pressure $q(\varphi, t)$ by the expression [11]

$$q(\varphi, t) \approx \frac{P(\cos\varphi - \cos\alpha)}{R_1(\alpha - ½\sin 2\alpha)},\quad P(t) = R_1 \int_{-\alpha}^{\alpha} q(\varphi, t)\cos\varphi\, d\varphi \tag{2.14}$$

If we introduce it into formulae (1.1), (1.3) and average the value $Q(\varphi, t)$ of the heat flux generated by dry friction over the contact area, we have

$$Q(t) = \omega_c R_1 f(T_*)\bar{q}(t),\quad \bar{q}(t) = \frac{P(\sin\alpha - \alpha\cos\alpha)}{R_1\alpha(\alpha - ½\sin 2\alpha)} \tag{2.15}$$

Now summarizing formulae (1.2) when $\omega = \omega_c$, and (2.5), (2.13) and (2.15) we write

$$\omega_c R_1 f(T_*)\bar{q} = Q_1 + \lambda_2(\epsilon R)^{-1}(T_* - D_0 - D_1\alpha^{-1}\sin\alpha) \tag{2.16}$$

Let us compute the deposit $\beta(t)$ of the ring with dry friction according to relations (1.13), (2.6) of [5] taking the temperature T_* in the contact area into account

$$\beta(t) = [\Delta(1 - \cos\alpha) - \epsilon BRT_*]\cos^{-1}\alpha \tag{2.17}$$

$$\epsilon\kappa R(R_1 G)^{-1} P_0 = [(\alpha_0 - ½\sin 2\alpha_0)(\Delta - \epsilon BRT_*)]\cos^{-1}\alpha_0$$

$$\beta_0 = \beta(0),\ \alpha_0 = \alpha(0),\ P_0 = P(0)$$

Furthermore, we take the limit value $\hat{\beta} = \hat{\beta}(t)$ $(\beta_0 < \hat{\beta} < h)$ for which the wear of the bushing caused by melting occurs and from the first formula of (2.17) we find the value of the angle $\hat{\alpha}$ corresponding to it. Knowing α_0 and $\hat{\alpha}$ we specify the required value of the critical speed of rotation of the shaft by formula (2.16) as $\omega_* = \max\omega_c(\alpha)$ $(\alpha_0 \leqslant \alpha \leqslant \hat{\alpha})$.

3. Let $\omega > \omega_*$. Then for the surplus amount of heat needed to melt the material of the bushing is given by

$$Q_0(\varphi, t) = \omega R_1 f(T_*) \bar{q}(\varphi, t) - Q_1 - \lambda_2(\epsilon R)^{-1}(T_* - D_0 - D_1 \alpha^{-1} \sin\alpha) \quad (|\varphi| \leqslant \alpha) \tag{3.1}$$

according to formulae (1.1)–(1.3), (2.5) and (2.13).

Now we take into account that [1, 2]

$$g\gamma \dot{v}(\varphi, t) = Q_0(\varphi, t) \quad (|\varphi| \leqslant \alpha) \tag{3.2}$$

where g is the specific heat of fusion of the material of the ring, and γ is its density. Basing ourselves on relations (3.1) and (3.2) we find

$$g\gamma v = R_1 f(T_*)\, \omega \int_0^t q(\varphi, \tau)\, d\tau - kt + \frac{D_1\lambda_2}{\epsilon R} J(t)$$

$$J(t) = \int_0^t \frac{\sin\alpha(\tau)}{\alpha(\tau)}\, d\tau, \quad k = Q_1 + \lambda_2(\epsilon R)^{-1}(T_* - D_0)$$

whence, according to (1.4) and (1.13), we obtain the following integral equation for evaluating the unknown contact pressure

$$a_1 q(\varphi, t) + a_2 \int_0^t q(\varphi, \tau)\, d\tau = [\beta(t) + \Delta]\cos\varphi - \Delta + \epsilon B T_* + mt - J(t) \tag{3.3}$$

$$(|\varphi| \leqslant \alpha(t))$$

Here the dimensionless variable $\tilde{t} = D_1\lambda_2(\epsilon R^2 g\gamma)^{-1} t$ and the notation

$$\tilde{q}(\varphi, \tilde{t}) = q(\varphi, t) G^{-1}, \quad \tilde{\beta}(\tilde{t}) = \beta(t) R^{-1}, \quad \tilde{\Delta} = \Delta R^{-1}, \quad \tilde{\alpha}(\tilde{t}) = \alpha(t),$$

$$\tilde{t} = \tilde{\eta}(\tilde{\alpha}), \quad m = \epsilon k R(D_1\lambda_2)^{-1}, \quad a_1 = \epsilon\kappa, \quad a_2 = \epsilon R R_1 f(T_*) G\omega(D_1\lambda_2)^{-1}$$

have been introduced. In (3.3) and in what follows the tilde is omitted.

In order to obtain the closed form of the original contact problem it is necessary to add to Eq. (3.3) the condition of quasi-equilibrium (the second formula of (2.14)) written in dimensionless form

$$N(t) = P(R_1 G)^{-1} = \int_{-\alpha}^{\alpha} q(\varphi, t)\cos\varphi\, d\varphi \tag{3.4}$$

and the relation

$$q(\varphi, t) = 0 \quad (|\varphi| > \alpha) \tag{3.5}$$

specifying the unknown area of contact of the shaft with the bushing.

Note that expression (3.5) enables us to rewrite the integral equation (3.3) as the system

$$Vq = [\beta(t) + \Delta]\cos\varphi - \Delta + \epsilon B T_* + mt - J(t) \tag{3.6}$$

$$Vq = a_1 q(\varphi, t) + a_2 \int_{\psi(\varphi)}^{t} q(\varphi, \tau)\, d\tau \quad (|\varphi| \leqslant \alpha, \ \psi(\varphi) \leqslant t \leqslant \hat{t})$$

$$\psi(\varphi) = \begin{cases} 0 & (|\varphi| \leqslant \alpha_0) \\ \eta(\varphi) & (\alpha_0 < |\varphi| \leqslant \alpha) \end{cases}$$

to solve which we use the algorithm from [5] together with the step-by-step method [12].

Let us divide the segment $[0, \hat{t}]$ into small sections $(t_i, t_{j+1}]$ $(i=0-(n-1),\ t_0=0,\ t_n=\hat{t})$ and substitute the approximate value

$$J(t)=\Sigma+\frac{\sin\alpha_i}{\alpha_i}t \tag{3.7}$$

$$\Sigma=\sum_{j=0}^{i}\left(\frac{\sin\alpha_{j-1}}{\alpha_{j-1}}-\frac{\sin\alpha_j}{\alpha_j}\right)t_j,\quad \alpha_{-1}=0$$

of the integral over the segment $(t_i, t_{i+1}]$ into the right-hand sides of Eqs (3.6).

We obtain a recurrent system of problems which enables us to find the basic characteristics of contact interaction at each time step. To do so, we first evaluate α_0 from formulae (2.17) (below we shall assume that $N(t)=N=\text{const}$ in (3.4)) and, in accordance with (3.7), we next write

$$Vq=[\beta(t)+\Delta]\cos\varphi-f_0(t),\ f_0(t)=\Delta-\epsilon BT_*+(\alpha_0^{-1}\sin\alpha_0-m)t$$

with $t\in(0, t_1]$ whence we find†

$$\beta(t)=\frac{f_0(t)-\Delta\cos\alpha}{\cos\alpha},\quad (a_1+a_2t)N=f_0(t)F(\alpha) \tag{3.8}$$

$$a_1q(\varphi,t)=\begin{cases} f_0(t)(\cos\varphi-\cos\alpha)\cos^{-1}\alpha+I(0,t) & (|\varphi|\leqslant\alpha_0)\\ I[\eta(\varphi),t] & (\alpha_0<|\varphi|\leqslant\alpha)\end{cases}$$

$$I(x,t)=-\theta\int_x^t f_0(\tau)\frac{\cos\varphi-\cos\alpha(\tau)}{\cos\alpha(\tau)}e^{-\theta(t-\tau)}d\tau,\quad \theta=\frac{a_2}{a_1}$$

$$\eta(\varphi)=\frac{F(\varphi)(\Delta-\epsilon BT_*)-a_1N}{a_2N-F(\varphi)(\alpha_0^{-1}\sin\alpha_0-m)},\quad F(\varphi)=\frac{\varphi-\frac{1}{2}\sin2\varphi}{\cos\varphi}$$

Setting $t=t_1$ in the second relation of (3.8) and evaluating the angle α_1 we transfer to the interval $(t_1, t_2]$ and so on. As the result, we have

$$Vq=[\beta(t)+\Delta]\cos\varphi-f_i(t),\ f_i(t)=\Delta-\epsilon BT_*+$$

$$+\Sigma+\left(\frac{\sin\alpha_i}{\alpha_i}-m\right)t,\quad t_i<t\leqslant t_{i+1} \tag{3.9}$$

for the ith $(i\leqslant1)$ step.

The solution of system (3.9) is given by (3.8) in which $f_0(t)$ should be replaced by $f_i(t)$ $(i\geqslant1)$, but for $\eta(\varphi)$ we take

$$\eta(\varphi)=\frac{F(\varphi)(\Delta-\epsilon BT_*+\Sigma)-a_1N}{a_2N-F(\varphi)(\alpha_i^{-1}\sin\alpha_i-m)}$$

†KOVALENKO Ye. V., Some contact problems for bodies with thin porous-elastic coating. Preprint No. 458, Inst. Probl. Mekh. Akad. Nauk SSSR, Moscow, 1990.

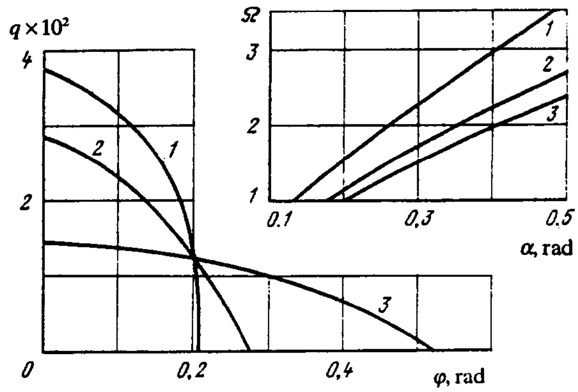

FIG. 2.

4. As an example, the basic characteristics of contact interaction during the melting of a sliding bearing were computed. The shaft and the race of the bearing are made of steel, the material of the bushing is the PTFE F4K20, the parameters take the following values: $R=13$ mm, $R_1=12.5$ mm, $R_2=14.5$ mm, $R_3=29.5$ mm, $l=50$ mm, $G=3\times10^2$ MPa, $\nu=0.4$, $f_0=0.1$, $f_2=0$, $\lambda_1=25.35$ W/(m °C), $\lambda_1=0.39$ W/(m °C), $\alpha_T=8\times10^{-5}$ (°C)$^{-1}$, $k_1=402$ W/(m °C), $k_2=36$ W/(m °C), $T_*=250$ °C, $D_0=235$ °C, and $D_1=4$ °C.

The variation of the critical speed of rotation of the shaft $\Omega=\omega_c(\alpha)/\omega_c(\alpha_0)$ for different values of the time-constant dimensionless load that acts upon the tenon is shown in Fig. 2 (the curves 1, 2 and 3 correspond to $N=3\times10^{-3}$, 7×10^{-3} and 10^{-2}). For the three cases in question we therefore have (1) $\alpha_0=0.135$, $\omega_c(\alpha_0)=7.96$ sec^{-1}; (2) $\alpha_0=0.178$, $\omega_c(\alpha_0)=4.51$ sec^{-1}; (3) $\alpha_0=0.201$, $\omega_c(\alpha_0)=3.55$ sec^{-1}. Note that the negative-valued depositions of the bushing $\beta_0R^{-1}=-0.479\times10^{-2}$, $\beta_0R^{-1}=-0.456\times10^{-2}$ and $\beta_0R^{-1}=-0.456\times10^{-2}$, correspond to specified values of the angles of contact. This fact shows that, as a rule, the wear caused by melting occurs in the bearing once there is a loss of the thermal-force stability [13]. For this reason the second term inside the parentheses of formula (1.13) predominates over the first term. Furthermore, setting $\hat{\beta}R^{-1}=0$ we evaluate the angle of contact $\hat{\alpha}=0.521$ which corresponds to this value of the deposit. Knowing this angle we find $\omega_*=29.99$ sec^{-1}, $\omega_*=12.85$ sec^{-1}, and $\omega_*=8.99$ sec^{-1} for the three cases of shaft loading considered.

The evolution of the dimensionless contact pressure with time in the process of wearing by melting with $N=10^{-2}$ and $\omega=9$ sec^{-1} is given on the left-hand side of Fig. 2 (curves 1, 2 and 3 correspond to $t=0$, 10^{-5} and 10^{-4}). Note that for the pressure the maximum discrepancy between the values presented above and those obtained by formula (2.14) is less than 15%. Therefore in order to obtain a rough estimate of the results, expression (2.14) may be used in practice which is much simpler than analogous relations (3.8).

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